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Water table fluctuations due to canal seepage and time varying recharge

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Abstract

Transient water table variation in an unconfined horizontal aquifer, lying between two canals located at different elevations above the impermeable barrier and receiving time varying recharge, is predicted by obtaining an analytical solution to the linearized Boussinesq equation. The proposed solution is for the generalized form of time varying recharge and has been obtained by applying an indigenously devised transformation to the flow governing equation. The solution for constant rate of recharge, a special case of the proposed solution, is compared with the existing analytical solution and almost identical values of water table rise are obtained. The effect of constant recharge, exponentially decreasing recharge and a combination of constant and exponentially decreasing recharge on the water table rise is analyzed with the help of a numerical example.

Keywords: Unconfined aquifer; Boussinesq equation; Water table; Canal seepage, Recharge

1. Introduction

Analytical solution to predict spatial and temporal variation of water table in an unconfined aquifer lying between two canals located at different elevations above the horizontal impermeable barrier was obtained by Gill (1984). Later, Mustafa (1987) incorporated a term of constant recharge in the linearized Boussinesq equation and obtained its solution to analyze the effect of constant recharge and seepage from canals on water table variation. Rai and Singh (1992) considered a variable rate of recharge (linearly decreasing with time which finally becomes constant) in the linearized Boussinesq equation and obtained the analytical solution by using the method of Laplace transformation to predict transient position of water table in the aquifer. Sewa Ram et al. (1994) also obtained the analytical solution for the same problem as considered by Mustafa (1987) but instead of employing Laplace transformation, a simple transformation was used. The results obtained from this solution were fairly close to Mustafa's solution.

In all these studies the rate of recharge was either zero or constant or linearly decreasing with time. However, Baumann (1952) reported that variation of the water table in response to recharge depends on the rate and duration of infiltration besides other factors like shape and size of recharge basin, hydraulic properties of aquifer, vertical distance between ground surface and initial water table. The dependence of water table variation on infiltration rate implies that

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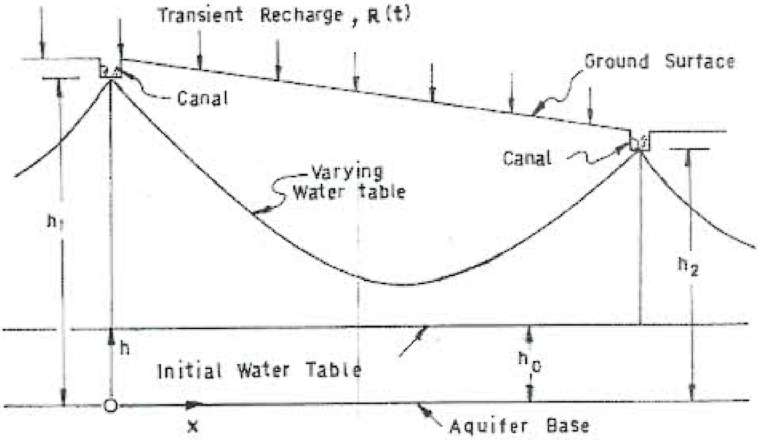

Fig. 1. Water table rise in an aquifer owing to canal seepage and time varying recharge.

the rate of recharge to the water table depends on the infiltration rate and decreases with time more or less in exponential form due to sediment clogging of soil pores beneath the recharge basin (Bear, 1979). In a field study in the Ghavin Plain of Iran, it has been shown that the solutions which are based on the assumption of constant rate of recharge cannot explain the observed recession of the water table (Zomorodi, 1991), thus requiring consideration of time dependent recharge. Abdulrazzak and Morel-Seytoux (1983) have also suggested that it would be of theoretical as well as of practical interest to consider time dependent recharge in order to simulate actual field conditions. Their experimental and analytical results show that the rate of recharge decreases exponentially with time. Later, Rai and Singh (1996) also advocated for consideration of time varying recharge. Manglik and Singh (1998) obtained mathematical solution to the two-dimensional equation and predicted water table fluctuations due to time varying recharge from a rectangular basin. Upadhyaya (1999) obtained several mathematical solutions for a number of physically identified flow situations to describe water table fluctuations in a sloping as well as horizontal aquifer in response to constant or time varying recharge and constant or depth dependent exfiltration. In the present study, the work of Mustafa (1987) has been extended by considering a time varying recharge rate approximately similar to the pattern of infiltration rate. The proposed analytical solution has been obtained using an indigenously devised transformation which is a simpler approach than the one in which Laplace transformation is used.

2. Problem definition

A definition sketch of water table rise in an aquifer owing to canal seepage and time varying recharge from land surface is shown in Fig. 1. The aquifer overlying a horizontal impermeable barrier receives time varying recharge between two canals located at elevations h_1 and h_2 above the impermeable barrier. The recharge rate is considered as exponentially decreasing with time from an initial value $R_1 + R_0$ to a lower value R_0 and thereafter remains constant. The assumptions considered for simplifying and formulating the physical problem in mathematical terms are: (1) the aquifer is homogeneous, isotropic and incompressible with time invariant hydraulic properties, and (2) the rate of recharge in comparison with the hydraulic conductivity is so small that the vertically percolated water flows almost horizontally after meeting the water table.

Flow is characterized by one-dimensional linearized Boussinesq equation derived using Dupuit's assumptions and Darcy's Law. This equation was used by Mustafa (1987), Rai and Singh (1992) and Sewa Ram et al. (1994) to describe movement of ground water in the flow system under consideration. The boundary value problem is given as:

$$\frac{\partial^2 z}{\partial x^2} + \frac{2R(t)}{K} = \frac{1}{a}\frac{\partial z}{\partial t} \tag{1}$$

$$z(x,0) = 0, \qquad 0 < x < L \tag{2}$$

$$z(0,t) = z_1, \qquad t > 0 \tag{3}$$

$$z(L,t) = z_2, \qquad t > 0 \tag{4}$$

where $z = h^2 - h_0^2$, $z_1 = h_1^2 - h_0^2$, $z_2 = h_2^2 - h_0^2$, h is the variable height of water table, h_0 the initial height of water table, $a = KD/f$, where K is the hydraulic conductivity, f the specific yield, D a constant of linearization and is approximated by $D = 0.5[h(t_e) + h_0(0)]$, where t_e the in-between period in which an estimate of h is made (D is taken as constant in this paper), x the space coordinate in horizontal direction, t the time of observation, L the spacing between two canals and $R(t) = R_0 + R_1 e^{-rt}$, where R_0 is constant rate of recharge, $R_0 + R_1$ the initial rate of recharge, r the decay rate of recharge and t the decay period of recharge.

3. Solution

A transformation to convert the linearized Boussinesq Eq. (1) into a heat flow equation is devised as:

$$z = v + \frac{2aR_0 t}{K} - \frac{2aR_1}{Kr}e^{-rt} \tag{5}$$

With this transformation, Eqs. (1)–(4) are transformed to:

$$\frac{\partial^2 v}{\partial x^2} = \frac{1}{a}\frac{\partial v}{\partial t} \tag{6}$$

with initial and boundary conditions as:

$$v(x,0) = \frac{2aR_1}{Kr} = f(x), \qquad 0 < x < L \tag{7}$$

$$v(0,t) = \left(z_1 - \frac{2aR_0 t}{K} + \frac{2aR_1 e^{-rt}}{Kr}\right) = f_1(t), \quad t > 0 \tag{8}$$

$$v(L,t) = \left(z_2 - \frac{2aR_0 t}{K} + \frac{2aR_1 e^{-rt}}{Kr}\right) = f_2(t), \quad t > 0 \tag{9}$$

The boundary value problem defined by Eqs. (6)–(9) is similar to heat flow equation with time dependent boundary conditions. The general solution to such boundary value problem reported by Ozisik (1980) is outlined as:

$$\begin{aligned} v(x,t) = {} & \frac{2}{L}\sum_{m=1}^{\infty} e^{-a\beta_m^2 t}\sin\beta_m x \int_0^L f(x')\sin\beta_m x'\,dx' \\ & + \left(1 - \frac{x}{L}\right)f_1(t) + \frac{x}{L}f_2(t) \\ & - \frac{2}{L}\sum_{m=1}^{\infty}\frac{\sin\beta_m x}{\beta_m} \\ & \times\left[f_1(0)e^{-a\beta_m^2 t} + \int_0^t e^{-a\beta_m^2(t-\tau)}\,df_1(\tau)\right] \\ & + \frac{2}{L}\sum_{m=1}^{\infty}(-1)^m\frac{\sin\beta_m x}{\beta_m} \\ & \times\left[f_2(0)e^{-a\beta_m^2 t} + \int_0^t e^{-a\beta_m^2(t-\tau)}\,df_2(\tau)\right] \end{aligned} \tag{10}$$

where $\beta_m = m\pi/L$ and $m = 1, 2, 3, \ldots, \infty$.

Substituting initial and boundary conditions as given by Eqs. (7)–(9) in Eq. (10) and some simplifications, the expression for $v(x, t)$ is obtained. Again, on substitution of expression for $v(x, t)$ in Eq. (5) the

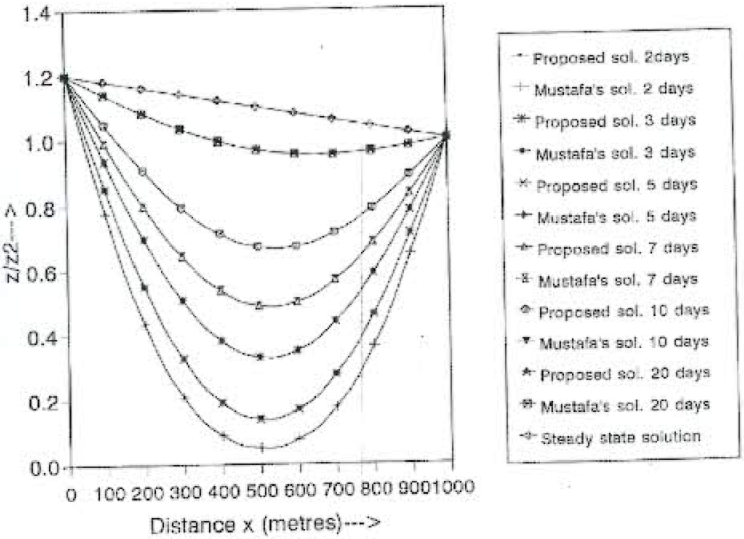

Fig. 2. Comparison of proposed and Mustafa's (1987) solution for water table rise due to seepage from canals in a horizontal aquifer.

analytical expression for $z(x, t)$ is obtained as:

$$z(x,t) = \left(1-\frac{x}{L}\right)z_1 + \frac{x}{L}z_2 - \frac{2}{L}\sum_{m=1}^{\infty}\frac{\sin\beta_m x}{\beta_m} \times (z_1-(-1)^m z_2)e^{-a\beta_m^2 t} + \frac{8a}{L}\sum_{m=1,3,5,\ldots}^{\infty}\frac{\sin\beta_m x}{\beta_m} \times\left[\left(\frac{R_1}{K}\right)\left\{\frac{e^{-rt}-e^{-a\beta_m^2 t}}{a\beta_m^2 - r}\right\}+\left(\frac{R_0}{K}\right)\left\{\frac{1-e^{-a\beta_m^2 t}}{a\beta_m^2}\right\}\right] \quad (11)$$

3.1. Special cases of the proposed analytical solution

(i) If a constant recharge, R_0 is assumed to occur in a horizontal aquifer, the solution for such a condition can be obtained by putting $R_1 = 0$ in Eq. (11) and written as:

$$z(x,t) = \left(1-\frac{x}{L}\right)z_1 + \frac{x}{L}z_2 - \frac{2}{L}\sum_{m=1}^{\infty}\frac{\sin\beta_m x}{\beta_m} \times (z_1-(-1)^m z_2)e^{-a\beta_m^2 t} + \frac{8a}{L}\left(\frac{R_0}{K}\right)\sum_{m=1,3,5,\ldots}^{\infty}\frac{\sin\beta_m x}{\beta_m}\left[\frac{1-e^{-a\beta_m^2 t}}{a\beta_m^2}\right] \quad (12)$$

Similarly a solution for constant rate of recharge, $R_0 + R_1$, can also be obtained by substituting $r = 0$ in Eq. (11). The solution given by Eq. (12) can be compared with Eq. (18) of Mustafa (1987) derived for the constant rate of recharge.

(ii) If a horizontal aquifer between two canals is subjected to an exponentially decreasing rate of recharge, the solution for such a condition can be obtained by putting $R_0 = 0$ in Eq. (11) and

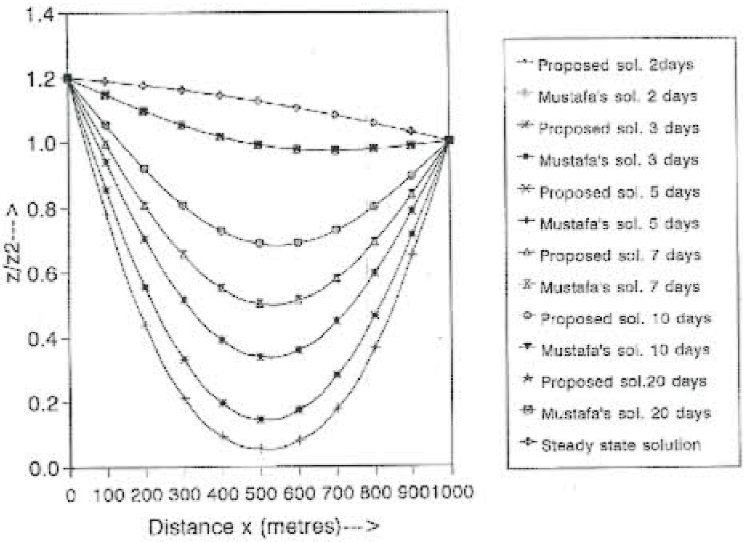

Fig. 3. Comparison of proposed and Mustafa's (1987) solution for water table rise due to seepage from canals and constant recharge from land surface in a horizontal aquifer.

written as:

$$z(x,t) = \left(1-\frac{x}{L}\right)z_1 + \frac{x}{L}z_2 - \frac{2}{L}\sum_{m=1}^{\infty}\frac{\sin\beta_m x}{\beta_m} \times (z_1 - (-1)^m z_2)e^{-a\beta_m^2 t} + \frac{8a}{L}\left(\frac{R_1}{K}\right)\sum_{m=1,3,5,\ldots}^{\infty}\frac{\sin\beta_m x}{\beta_m} \times\left[\left\{\frac{e^{-rt}-e^{-a\beta_m^2 t}}{a\beta_m^2 - r}\right\}\right] \quad (13)$$

(iii) Steady state solution of Eq. (1) can be obtained by substituting $\partial z/\partial t = 0$ to the right-hand side of Eq. (1) and R_0 in place of $R(t)$ because at $t=\infty$, e^{-rt} becomes zero. The solution of the equation for the boundary conditions $z(0) = z_1$ and $z(L) = z_2$ is obtained as:

$$z(x) = z_1\left(1-\frac{x}{L}\right) + z_2\frac{x}{L} + \frac{R_0}{K}x(L-x) \quad (14)$$

Eq. (14) gives the steady-state water table profile in a horizontal aquifer receiving constant recharge, R_0. If there is no recharge from the land surface, the steady state solution for ultimate water table rise due to seepage from canals only, which may be obtained by putting $R_0 = 0$ in Eq. (14). The solution given by Eq. (14) is similar to the one obtained by Mustafa (1987) for prediction of a steady state water table profile between two canals.

4. Results and discussion

The effect of constant, exponentially decreasing and a combination of constant and exponentially decreasing recharge on water table fluctuations in the aquifer was studied by considering the example given by Mustafa (1987). In the numerical example, the values of various parameters are: $L = 1000$ m, $z_0 = 0$, $z_1 = 120$ m^2, $z_2 = 100$ m^2, $R_0/K = 1\times10^{-5}$, $R_1/K = 2\times10^{-5}$, $a = 12\,000$ m^2 day^{-1}, and the values of 'r' as 0, 0.05, and 0.10 day^{-1}. In order to

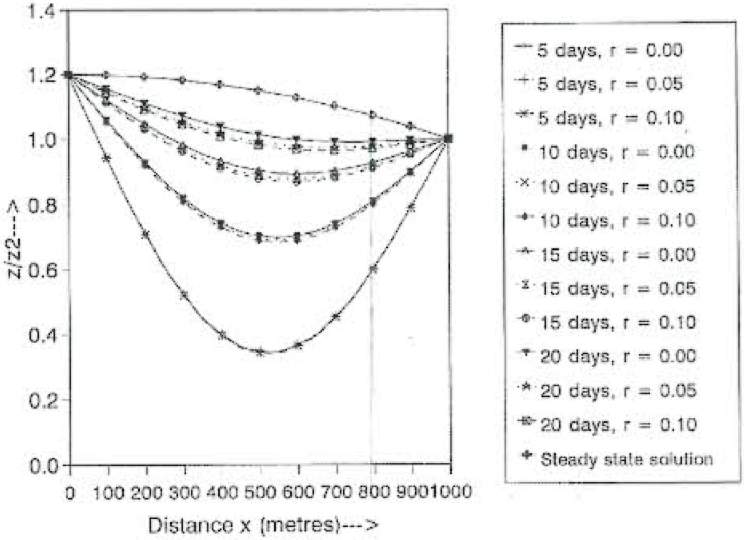

Fig. 4. Effect of decay constant 'r' on water table rise between two canals due to exponentially decreasing recharge from land surface in a horizontal aquifer.

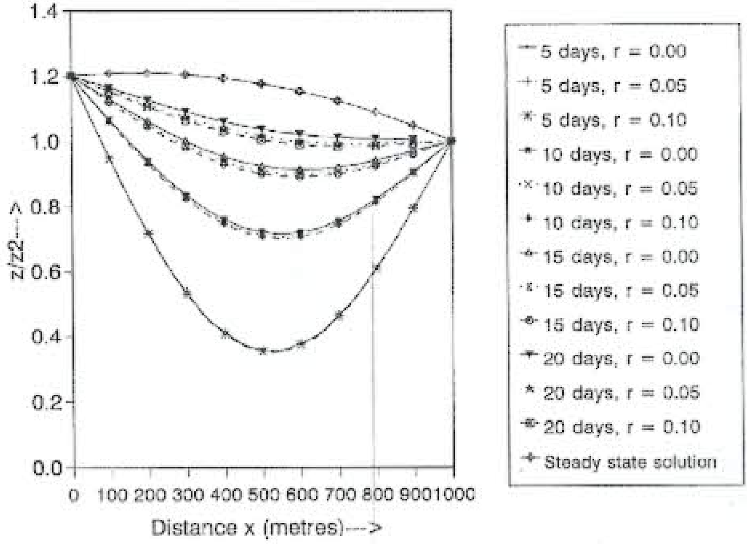

Fig. 5. Effect of decay constant 'r'' on water table rise between two canals due to a combination of exponentially decreasing and constant recharge from land surface in a horizontal aquifer.

compare the results of the proposed solution with Mustafa's solution corresponding to $R_0/K = 0$ and 1×10^{-5}, and $R_1/K = 0$, the values of $z(x, t)$ were computed at distances $x = 100$, 200, 300, …, 1000 m from the origin for the time periods $t = 2$, 3, 5, 7, 10, and 20 days. Effect of decay constant, 'r', on water table rise was studied for the time periods $t = 5$, 10, 15, and 20 days. The values of water table elevation were found to converge for a number of terms ($m = 5$), so that only five terms were retained in the solution.

The plots of z/z_2 as computed from the proposed analytical solution and Mustafa's solution for the case of zero recharge and constant recharge are shown in Figs. 2 and 3, respectively. It may be observed from the figures that values of water table elevation computed from both solutions for zero and constant recharge are almost identical, which partially validates the correctness of the proposed solution. Effect of constant recharge from the land surface on water table rise may be observed by comparing the water tables corresponding to the same time level in Figs. 2 and 3. The comparison shows that the rise in water table is faster when constant recharge is considered compared to the case of no recharge. Steady state water table rise with zero and constant recharge is also shown in Figs. 2 and 3, respectively. It may be observed from Fig. 2 that nondimensional steady state water table (z/z_2) decreases linearly from the canal at higher elevation to the canal at lower elevation, whereas an asymmetric water mound between two canals on account of constant recharge may be observed in Fig. 3.

The values of z/z_2 were plotted against x for the time periods $t = 5$, 10, 15, 20 days to study the effect of constant recharge, R_1, with $r = 0$ and exponentially decreasing recharge, R_1, with $r = 0.05$ and 0.10 on water table rise in the horizontal aquifer and are shown in Fig. 4. It may be observed from Fig. 4 that water table rise depends on the decay constant, 'r', associated with R_1 and with increase in the value of decay constant, the rise of water table slows down. Steady state water table rise due to nondimensional constant recharge, $R_1/K = 2 \times 10^{-5}$, is also given in Fig. 4, which again shows water stagnation on the land surface between two canals in the form of an asymmetric mound.

The plot of z/z_2 against x for the time period $t = 5$, 10, 15, 20 days, considering time varying recharge, $R(t) = R_0 + R_1 e^{-rt}$ with $r = 0.0$, 0.05, 0.10, and for a steady state condition corresponding to constant recharge, $R_0 + R_1$, is shown in Fig. 5. Comparison of water table heights corresponding to $r = 0$ in Figs. 4 and 5 shows that steady or transient rise of water table in Fig. 4 is slower than that in Fig. 5 because the former water table rise is for constant recharge R_1 whereas the later water table rise is for constant recharge $R_0 + R_1$. It may be observed from both Figs. 4 and 5 that the water table rise reduces due to consideration of exponentially decreasing or a combination of constant and exponentially decreasing recharge, and the magnitude of reduction, which is more in the central part of the aquifer, increases with time.

5. Conclusions

An analytical solution of the linearized Boussinesq equation incorporating time varying recharge is obtained by employing an appropriate and simple transformation and used to predict the spatial and temporal variation of water table in a horizontal aquifer due to canal seepage and time varying recharge for a selected numerical example. Time varying recharge of the form, $R(t) = R_0 + R_1 e^{-rt}$, seem to be a more generalized and practical form of recharge for which water table rise between two canals has been analyzed. Results reveal that the water table rise in a horizontal aquifer lying between two canals decreases due to consideration of exponentially decreasing or a combination of exponentially decreasing and constant recharge, and the magnitude of reduction, which is more in the central part of the aquifer, increases with time.

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